

# Radiation Impedance of Insertion Devices and the Impact on the Beam Instability in Damping Rings

*J.H. Wu, T.O. Raubenheimer, G.V. Stupakov*  
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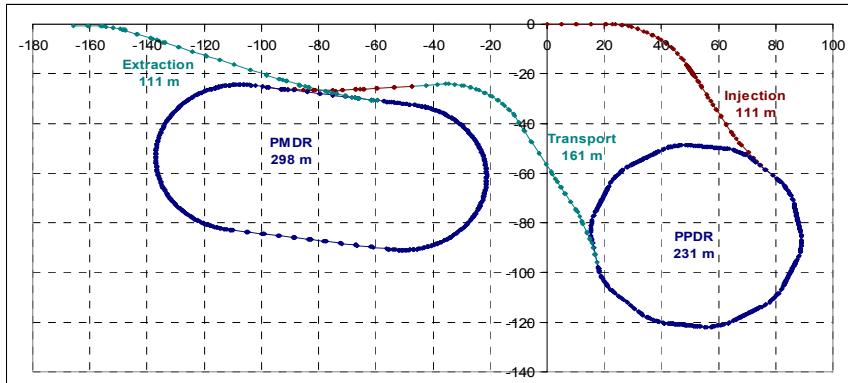
# Motivation

- Damping rings for future Linear Colliders, e.g., NLC, requires very fast damping → long wigglers;
- Extracted beam must be very stable → sensitive to bursting instabilities;
- Microwave instabilities from vacuum system are real concerns;
- Recent CSR studies indicate additional potential problem.

# Damping Ring Nominal Parameters

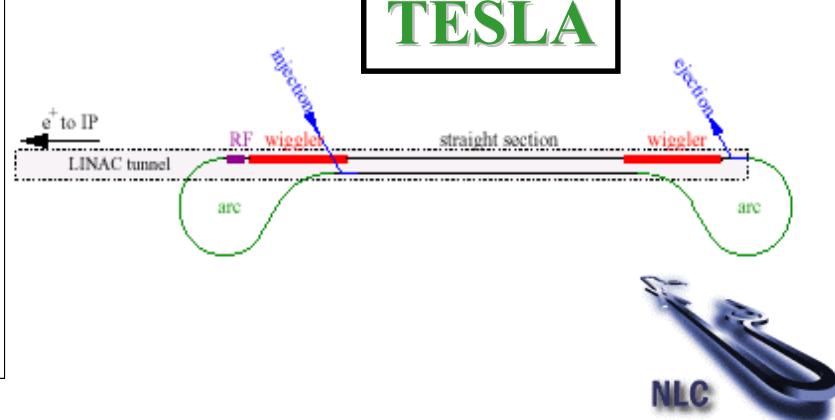
	NLC/JLC	ATF	ALS	TESLA	LEP
circumference / km	0.3	0.14	0.2	17	26
E / GeV	1.98	1.3	1.9	5	46
R / m	5.5			80	
v / e-4	9.09			9	
$\alpha$ / e-4	2.95			1.2	
wiggler length / m	46	8	6?	432	
wiggler K	54			56	
particle / bunch/e+10	0.75			2.0	
bunch length /mm	3.6			6.0	

**NLC**



**S**tanford  
**L**inear  
**A**ccelerator  
**C**enter

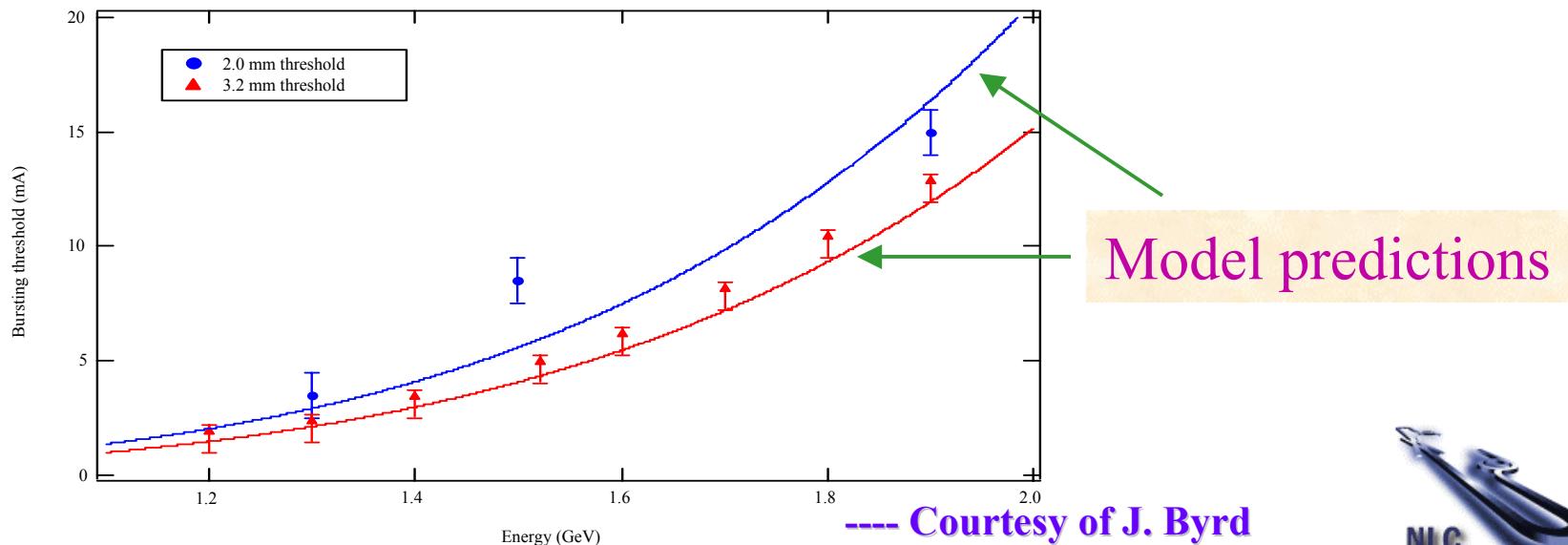
**TESLA**



**NLC**

# CSR instability

- Stupakov-Heifets theory [PRST-AB, 5(2002)054402] indicates a potential instability due to the CSR in dipoles;
- Experimental observations [John Byrd, *et al.*, EPAC 2002, P. 659; NSLS/BNL; BESSY-II]



# CSR Impedance

- Theory is developed [Talk of Stupakov]
- We need impedance
  - Dipoles ( $\alpha > 0$ ) (known, [Murphy, *et al.*, 1995])
    - Scaling  $Z(k) = -i A k^{1/3} / R^{2/3}$ 
$$A = 3^{-1/3} \Gamma(2/3)(\sqrt{3} i - 1)$$
    - longest wavelength determines threshold
  - Wigglers ( $\alpha < 0$ ) (We need to solve)

# How to treat wiggler (An estimate)

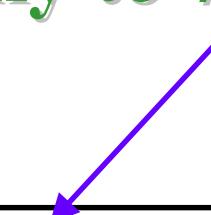
- Treat the wiggler as  $2N$  pieces of Dipoles

$$R(z)^{-1} = \frac{k_w K |\cos(k_w z)|}{\gamma} \text{ average } \bar{R}^{-1} = \frac{2}{\pi} \frac{k_w K}{\gamma}$$

- Impedance

$$Z(k) = iA \left\{ \frac{k^{1/3}}{(R_D)^{2/3}} \frac{L_D}{C} + \frac{k^{1/3}}{(\bar{R}_W)^{2/3}} \frac{L_W}{C} \right\}$$

- Threshold in NLC Damping Ring drops from  $1.75 \times 10^{10}$  with dipoles only to  $7.25 \times 10^9$ .



**It is serious!!! Recall the design value is  $7.5 \times 10^9$ ;**

**But is bending magnet approach for wiggler valid?**

# Length Scales

- Cooperative/formation length for a radiation wavelength  $\lambda_f$ 
  - Dipole:  $L_f \sim \sqrt[3]{24R^2\lambda_f}$
  - Wiggler:  $\lambda_f \sim \lambda_{\text{FEL}} \sim 13 \mu\text{m}$       Periodicity leads to interference
- Pipe cut-off wavelength: 1 mm;
- Long-range wake applicable for:  $\lambda \gg \lambda_{\text{FEL}}$

For  $\lambda < \lambda_{\text{FEL}}$  → “may” approximate as 2N dipoles ;  
but for longer, has to include interference; need  
look at FEL frequency and harmonics

# CSR Wiggler Wake

- Early studies [Yong-Ho Chin, LBL-29981 (1990); Saldin *et al.*, NIMA 417 (1998) 158]
- We introduce Wake as

$$W(s) = -k_w \int_{-\infty}^s ds' G(s-s') \frac{d\lambda(s')}{ds'}$$

therefore, the wake Green function

$$w(s) = -k_w \frac{dG(s)}{ds}$$

- Our results are universal by expressing G with only one parameter  $\varsigma = \frac{\gamma^2 k_w}{K^2} s$ , as long as K is large.

# Calculation details

- **The rate of energy loss**

$$\hat{s} = \gamma^2 k_w s$$

$$\frac{d\varepsilon}{cdt} = e^2 k_w \int_{-\infty}^s ds' D(\hat{s} - \hat{s}', K, \hat{z}) \frac{d\lambda(s')}{ds'}$$

$$\hat{z} = k_w z$$

**where**

$$D(\hat{s} - \hat{s}', K, \hat{z}) = \frac{1}{\hat{s}} - 2 \frac{\Delta - K^2 B(\Delta, \hat{z}) [\sin \Delta \cos \hat{z} + (1 - \cos \hat{z}) \sin \hat{z}]}{\Delta^2 + K^2 B^2(\Delta, \hat{z})}$$

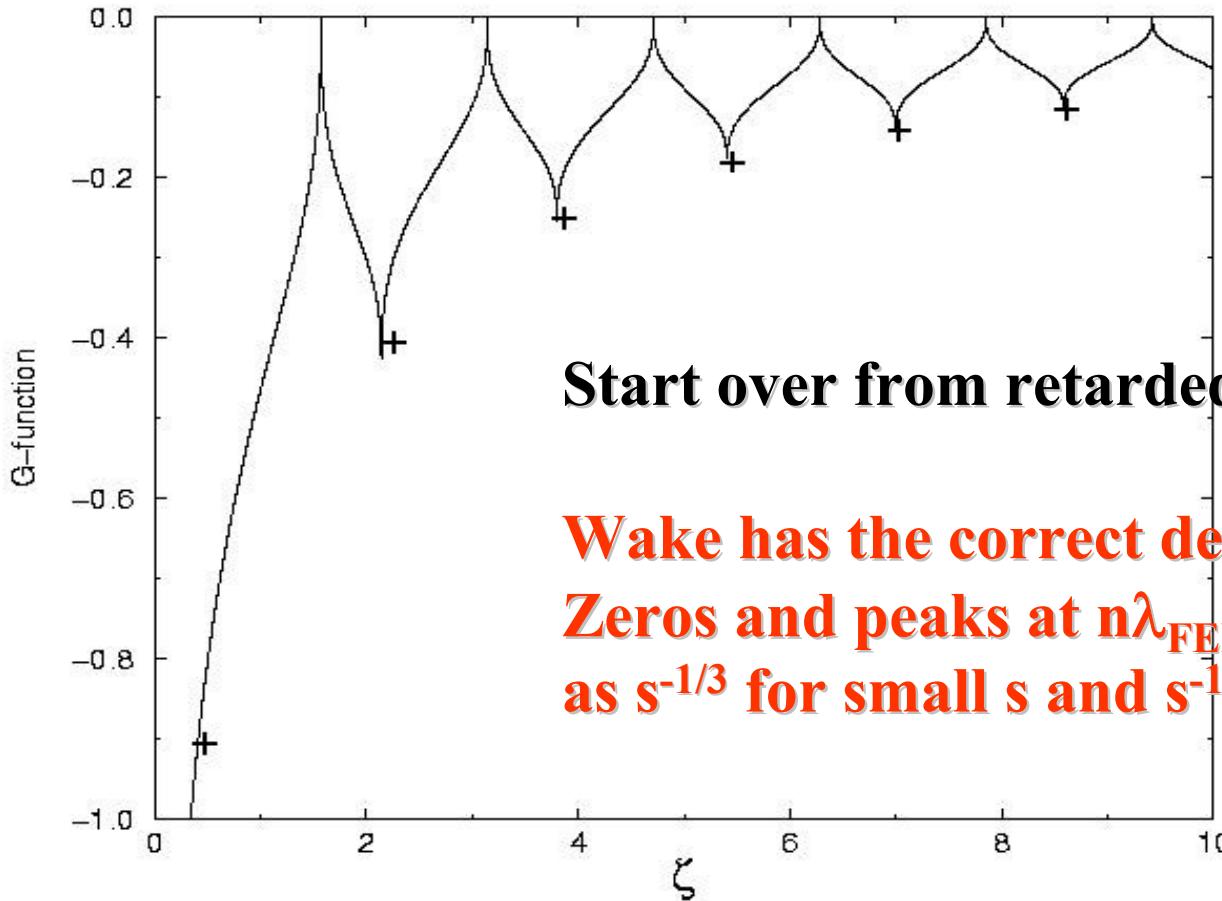
$$B(\Delta, \hat{z}) = (1 - \cos \Delta - \Delta \sin \Delta) \cos \hat{z} + (\Delta \cos \Delta - \sin \Delta) \sin \hat{z}$$

**and  $\Delta$  is determined by**

$$\hat{s} = \frac{\Delta}{2} \left( 1 + \frac{K^2}{2} \right) + \frac{K^2}{4\Delta} \{ [2(1 - \cos \Delta) - \Delta \sin \Delta] \times (\cos \Delta \cos 2\hat{z} + \sin \Delta \sin 2\hat{z}) - 2(1 - \cos \Delta) \}$$

$$G(s) = \frac{1}{\pi} \int_0^\pi d\hat{z} D(\hat{s}, K, \hat{z})$$

# Numerical G-function



Start over from retarded potential

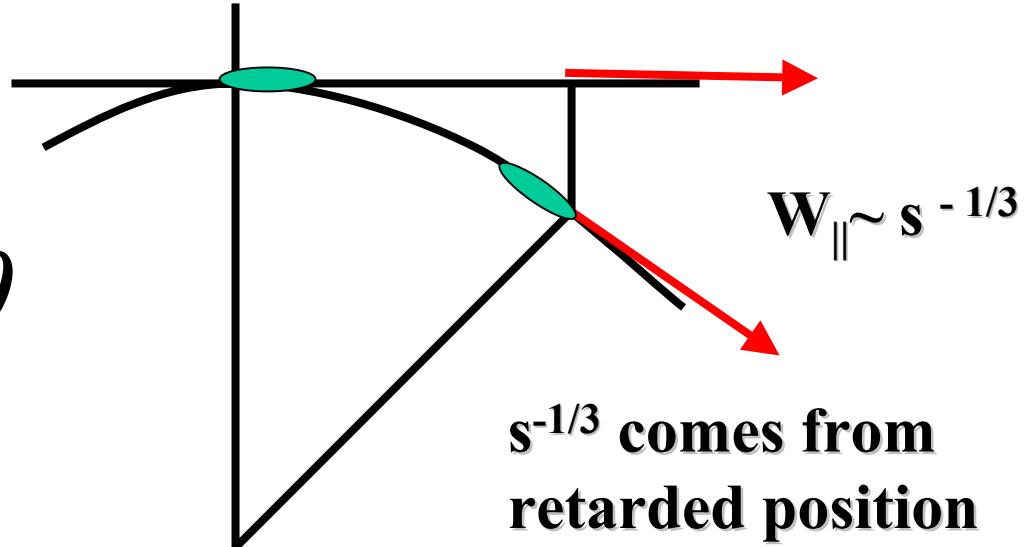
Wake has the correct dependences:  
Zeros and peaks at  $n\lambda_{\text{FEL}}/2$  and scaling  
as  $s^{-1/3}$  for small  $s$  and  $s^{-1}$  for large  $s$

# Simple Physics Model

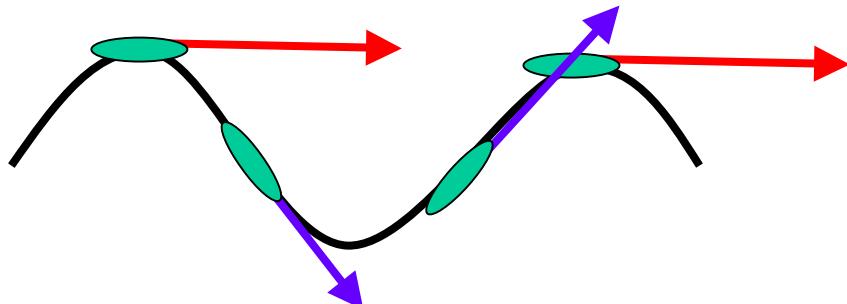
(ala Derbenev)

- Dipole:

$$F_{\parallel} = eE_{\perp} \sin\theta$$



- Wiggler:



- $W_{\parallel} = 0$ , for  $s = n \lambda_{\text{FEL}}$ ;
- $W_{\parallel} = \text{max}$ , for  $s = \lambda_{\text{FEL}}/2$ ;
- $W_{\parallel} \sim s^{-1/3}$ , for  $s \rightarrow 0$ ;
- $W_{\parallel} \sim s^{-1}$ , for  $s \gg \lambda_{\text{FEL}}$ .

# Analytical check

- Estimate of the extreme points

$$G(\zeta) = \begin{cases} 0; & \text{for } \zeta = \frac{n\pi}{2} \\ -\frac{4(2n+1)\pi}{4 + [(2n+1)\pi]^2}; & \text{for } \zeta \approx \frac{(2n+1)\pi}{4} - \frac{1}{(2n+1)\pi} \end{cases}$$

with  $n=1,2,\dots$

with  $n=0,1,\dots$

They are plotted as “+”s in the plot for G.

# Analytical check

- Short-range limit [perturbative approach]

$$G(\varsigma) = -\frac{4 \cdot 3^{2/3} \Gamma\left(\frac{11}{6}\right)}{5\sqrt{\pi} \Gamma\left(\frac{4}{3}\right)} \varsigma^{-1/3}$$

- Simple approach

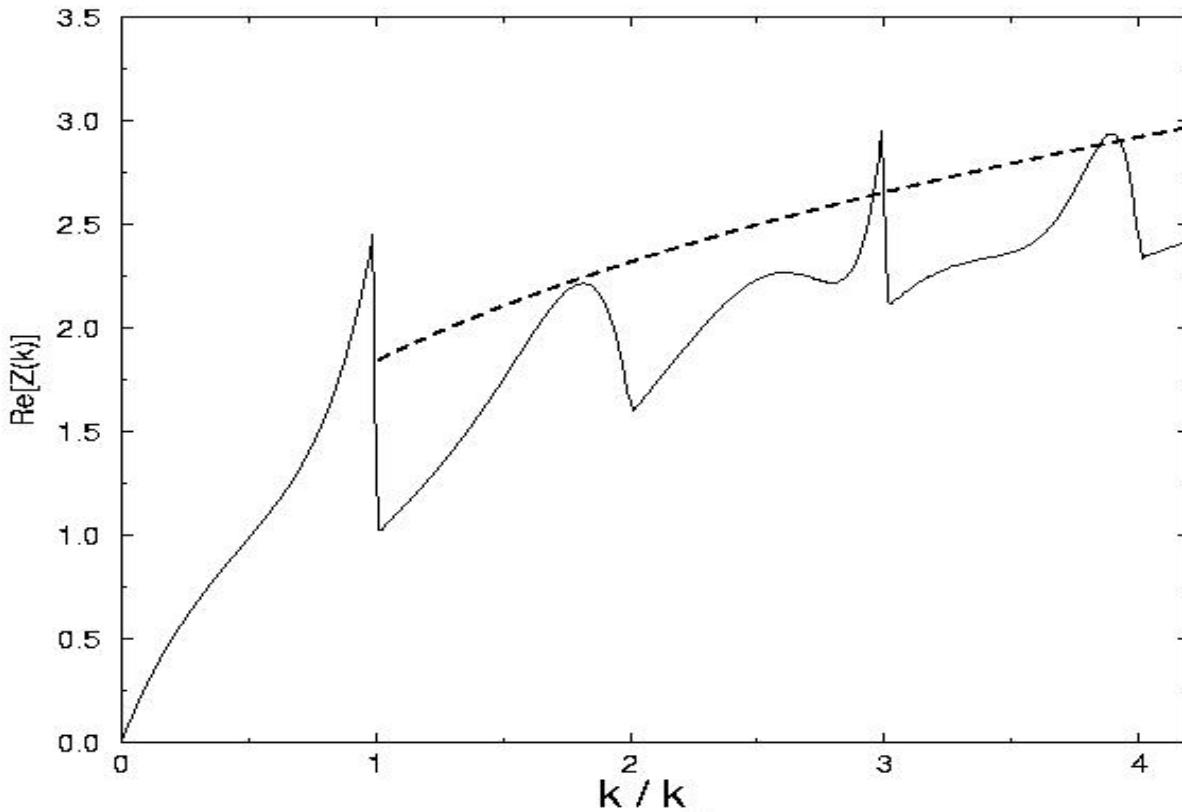
$$\hat{z} = k_w z$$

**Dipoles**  $G(\varsigma) = -\frac{2}{(3R^2)^{1/3}} \varsigma^{-1/3}$

**Effective radius**  $\frac{1}{R(\hat{z})^2} = \frac{k_w^2 K^2 \cos^2 \hat{z}}{\gamma^2}$

$$\Rightarrow \frac{1}{\pi} \int_0^\pi d\hat{z} \cos^{2/3} \hat{z}$$

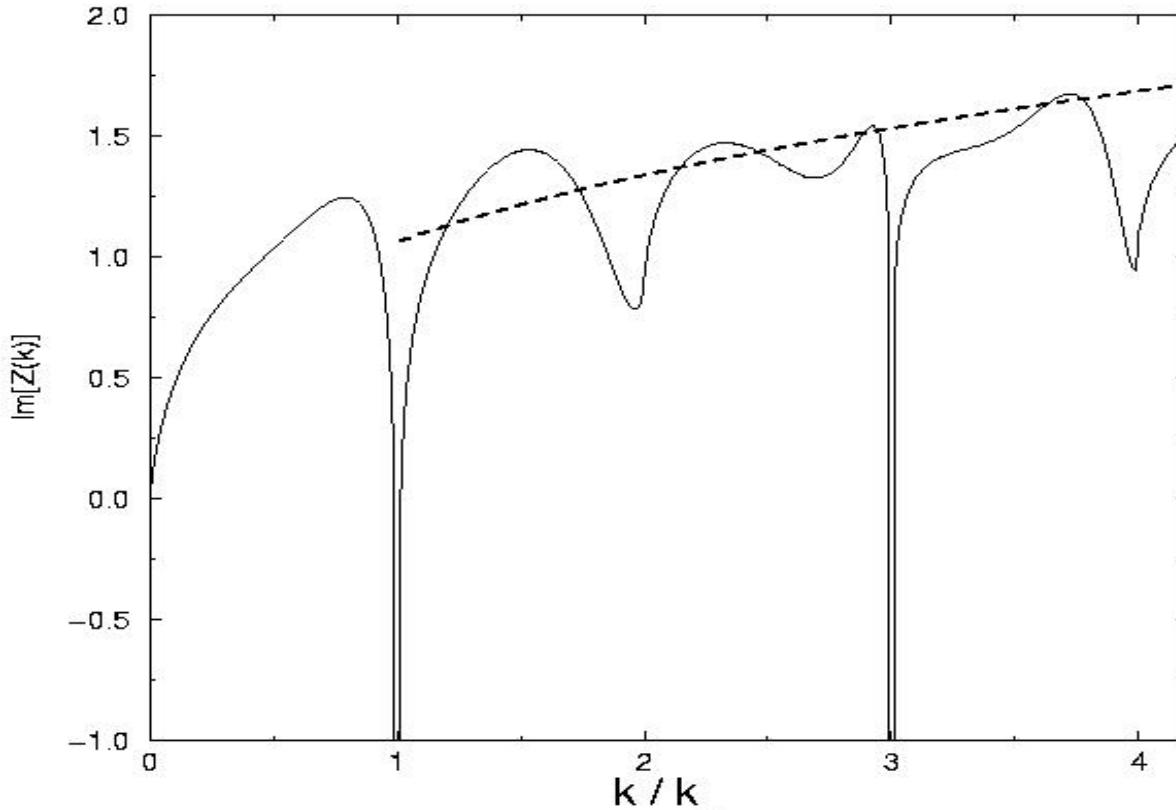
# Real Part of the Impedance



Real part of the impedance with peaks at  $n\lambda_{\text{FEL}}$  which continues to increase towards  $\lambda_c$ . Check with the wiggler radiation spectrum [Alferov, *et al.*, 1974; Krinsky, *et al.*, 1983], perfect agreement.

$$k_- = \frac{4k_w\gamma^2}{K^2} \quad \text{FEL wavenumber}$$

# Imaginary Part of the Impedance



$$k_- = \frac{4k_w\gamma^2}{K^2} \quad \text{FEL wavenumber}$$

# Analytical Asymptotic Expression

- Low-frequency (up to  $k < 0.1k_{\text{FEL}}$ )

$$Z(k) = \frac{K^2}{\gamma^2} \left( \frac{\pi}{4} k - \frac{i}{2} \underbrace{k \log[\frac{k}{k_{\text{FEL}}}]}$$

Real part of the impedance is linear in  $k$ ;  
 Imaginary part is  $k \log[k]$ .

with the FEL wavenumber.

$$k_{\text{FEL}} = \frac{4\gamma^2 k_w}{K^2}$$

- High-frequency ( $k > k_{\text{FEL}}$ )

$$Z(k) = -i \frac{6 \Gamma\left(\frac{11}{6}\right)}{5\sqrt{\pi} \Gamma\left(\frac{4}{3}\right)} A k^{1/3} \left( \frac{K^2}{\gamma^2 k_w^2} \right)^{1/3}$$

$$A = 3^{-1/3} \Gamma(2/3)(\sqrt{3} i - 1)$$

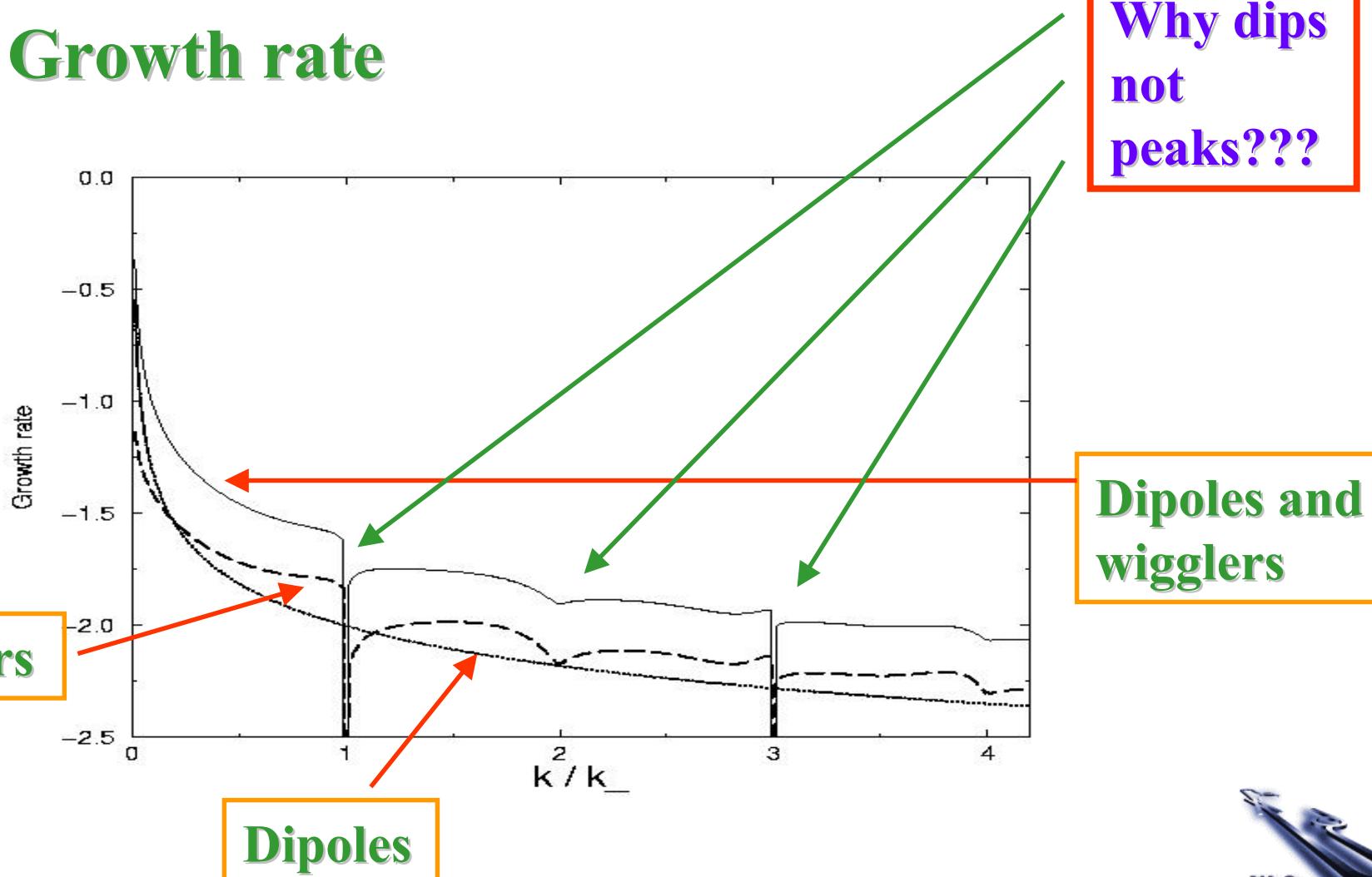
Effective  
dipole radius

Average over one period

# Complete Numerical Results

## NLC damping ring

- Growth rate

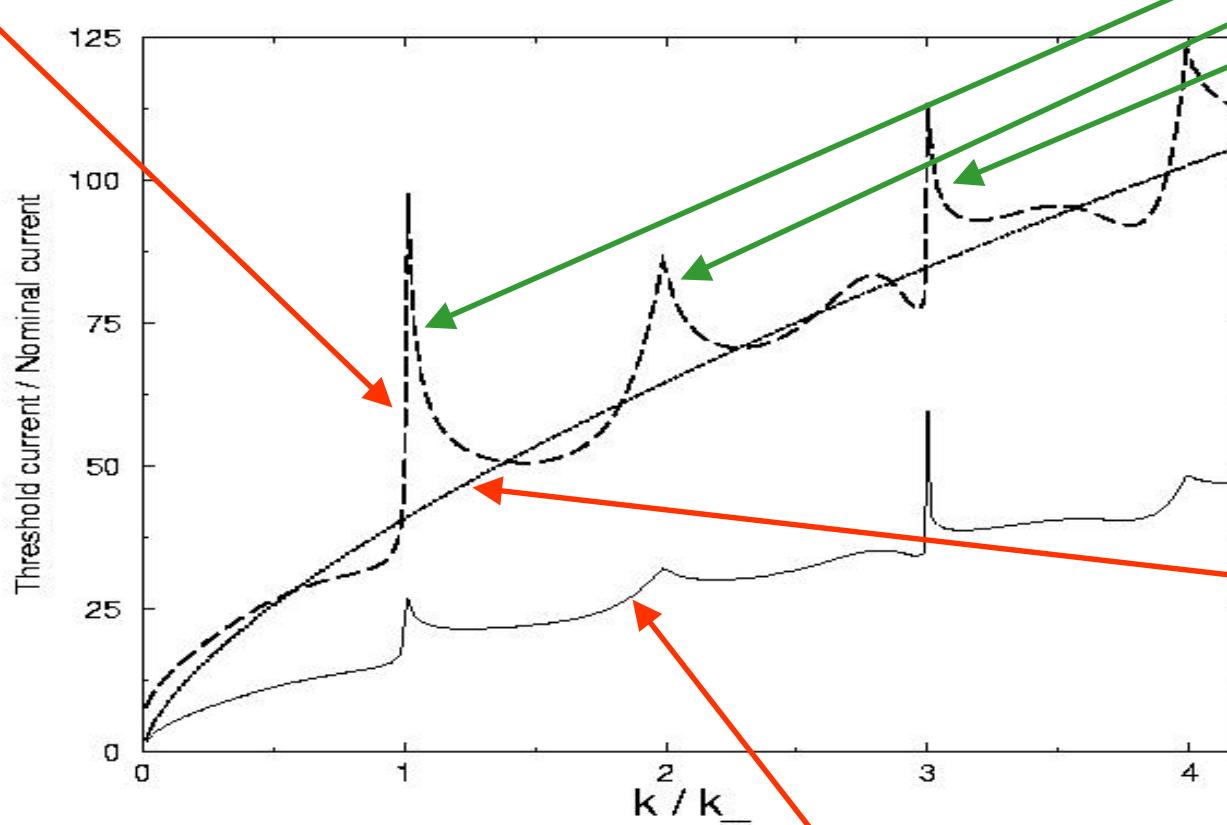


# Threshold

## NLC damping ring

W wigglers

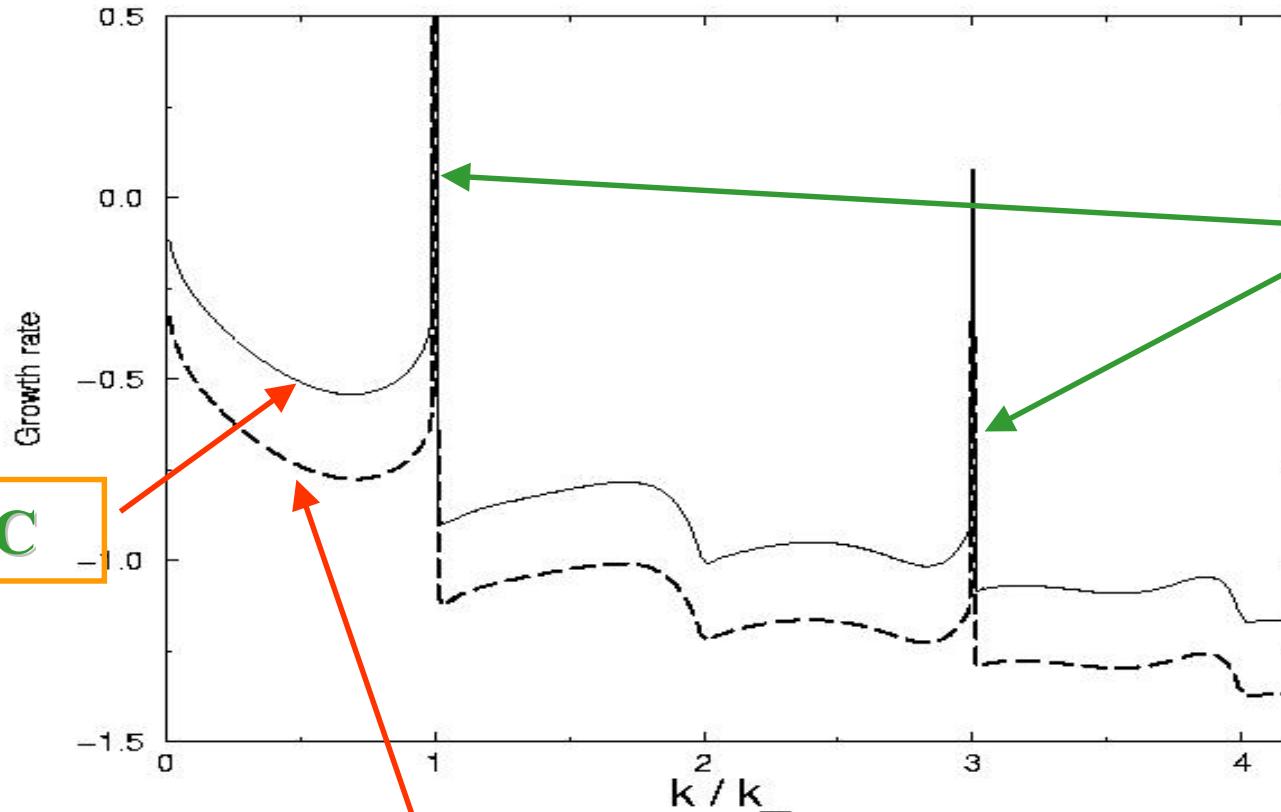
Why  
peaks  
not  
dips???



Dipoles

Dipoles and wigglers

# Inside Wiggler



# Estimate at FEL frequency

- At FEL and harmonics,  $Z = R - i L$ , with  $L \gg 1$ ,  $R > 0$ , but  $R \ll L$ .
- Gaussian in energy spread  $\rho_0 = \frac{n_0}{\sqrt{2\pi}\delta_0} e^{-\delta^2/(2\delta_0^2)}$
- Dispersion relation is

$$1 = - \frac{i \Lambda Z(k)}{\sqrt{2\pi} k} \int \frac{dp}{\Omega \pm p} \frac{p e^{-p^2/2}}{}$$

$$\approx \pm \frac{i \Lambda Z(k)}{\Omega^2 k} \approx \pm \frac{\Lambda L}{\Omega^2 k}$$

 $\text{Im}(\Omega) \approx 0$   
 for  $\alpha > 0$

# Low-frequency Impedance

## Analytical

- **Wiggler impedance:**

$$Z_W(k) = \frac{K^2}{\gamma^2} \left\{ \frac{\pi k}{4} - \frac{i k}{2} \text{Log} \left[ \frac{k}{k_{\text{FEL}}} \right] \right\}$$

- **Total impedance:**

$$Z(k) = \left\{ Z_D(k) \frac{L_D}{C} + Z_W(k) \frac{L_W}{C} \right\}$$

	Nominal ( $10^{10}$ )	Threshold from full-impedance ( $10^{10}$ )	Threshold from low-frequency ( $10^{10}$ )
NLC	0.75	1.3	1.3
TESLA	2.0	78.8	

We may use the low-frequency impedance for practical purpose!

# Discussion

- Due to the impedance of the dipole and that of the wiggler have different scaling at low-frequency, one may make use of it in making a Coherent Light Source [J. Byrd, *et al.*, 2001; Murphy, *et al.*, 1994] .
- For practical CSR purpose, ( $< 0.1 f_{FEL}$ ), the wiggler impedance is

$$Z_W(k) = \frac{K^2}{\gamma^2} \left\{ \frac{\pi k}{4} - \frac{i k}{2} \text{Log} \left[ \frac{k}{k_-} \right] \right\}$$

- What about waveguide cut-off and resonances?
- How to compare with experiment?

# An estimate on the FEL instability

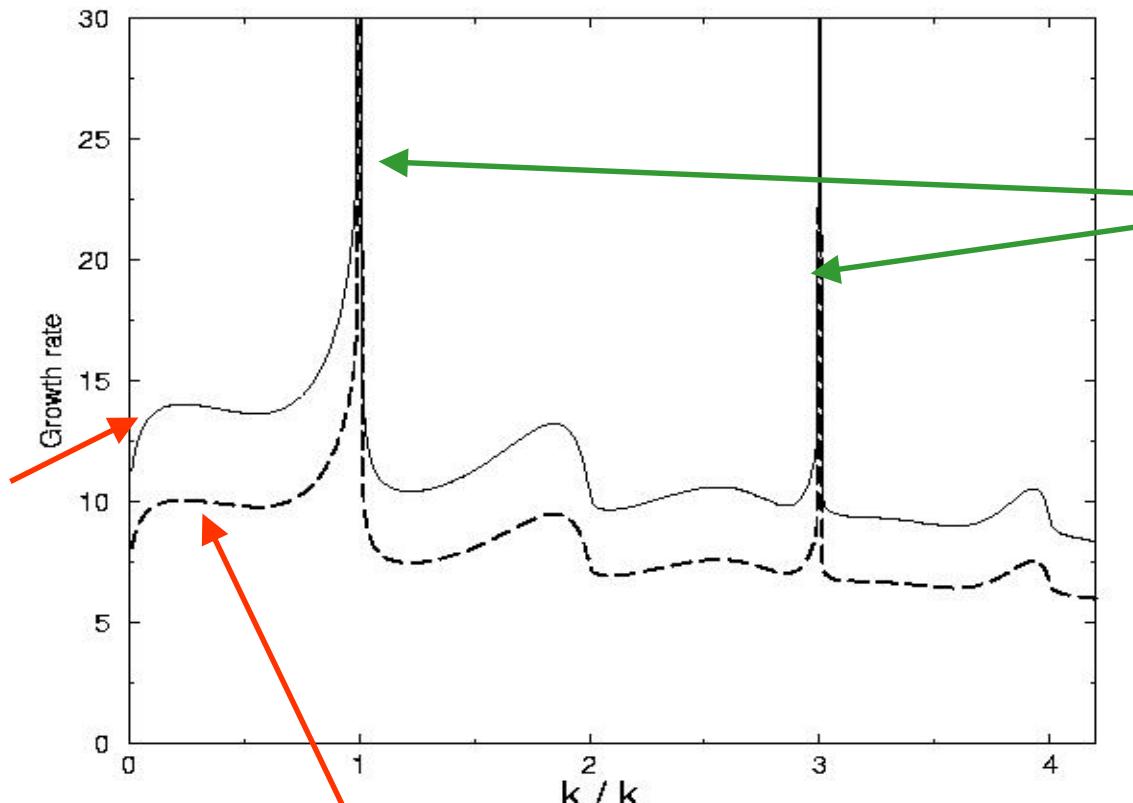
- 1-D theory predicts very short-gain length
- However, minimal beam-radiation overlap
- 3-D TDA simulation gives, for NLC wiggles,

$$L_G^{wiggler} \approx 31 \text{ m}$$

but the wiggler is only about 50 m.

Single pass isn't dangerous, but how about multi-turn? Need numerical simulations with momentum compaction for ring.

# Undulator (X-ray FEL)



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